Worked Example for Factor Analysis

This example uses the ASA software integrated into Excel or SPSS (www.asastat.com). ASA is, in part, a point-and-click interface to R but analyses can be conducted from within SPSS or Excel. All data are hypothetical. We assume you have read the primer on factor analysis.

Measures of the occurrence of 9 common syndrome clusters associated with posttraumatic stress disorder (PTSD) were obtained for a sample of war veterans who suffered high levels of stress during war. The symptoms assessments were the occurrence of (1) upsetting memories from the past event, (2) nightmares, (3) intense reactions to reminders of the event, (4) avoiding activities that might remind one of the event, (5) loss of interest in activities and life in general, (6) feeling emotionally numb, (7) excessive worrying, (8) difficulty concentrating, and (9) feeling jumpy and easily startled. The frequency/severity of each symptom category was assessed with multiple items and averaged within a category, with scores ranging from 0 to 10 for each category. Higher scores indicated the symptom category occurred more frequently and was more problematic in the veterans' lives. The variables were labeled ptsd1 through ptsd9 in the data set.

The symptom categories were correlated with one another and we want to test if the correlations among the categories can be accounted for by latent factors reflecting generalized stress reactions. We decide to conduct an exploratory factor analysis of the nine symptom categories. We make an initial decision to use maximum likelihood extraction because it has a strong underlying statistical theory. We plan to use an oblique rotation, geomin, because it tends to be among the better rotation methods available (see the primer on factor analysis for details).

The ASA software routinely reports confidence intervals for key parameters in statistical models. There are different ways of presenting confidence intervals. One strategy is to report them directly. Another strategy is to report them as margins of error, much like the margins of error you see for political polls on television or in print media. In this case, one calculates the half width of the confidence interval and reports it in "plus or minus" format. For example, in a political poll, you might be told that the percent of people endorsing a candidate is $50\% \pm 5\%$. In this case, the confidence interval is 45% to 55%. This is an efficient way of summarizing the interval. In some cases, confidence

intervals are asymmetric. When this occurs, some researchers will report the lower and upper margin of error separately. Alternatively, the researcher might calculate the absolute difference between the lower limit and the parameter estimate as well as the absolute difference between upper limit of the interval minus the parameter estimate and then report whichever difference is larger using the \pm format. Some analysts prefer the use of credible intervals in Bayesian analytic frameworks instead of confidence intervals for characterizing margins of error (see Curran, 2005).

PRELIMINARY ANALYSES

The first step in the analysis is to gain a sense of the distributions of the variables and to determine if issues with outliers, non-normality, and model misspecification will likely arise. We do not present these analyses here (see the ASA software for worked examples of them), but all was in order, so we proceed accordingly.

THE FACTOR ANALYSIS

The Number of Factors

Our first task is to determine the number of factors needed to account for the correlations between symptoms. We approach the matter from multiple perspectives. We first conduct an analysis for a single factor model to obtain traditional statistics typically relied upon by researchers for choosing factors. Here is the factor information for the correlation matrix as focused on eigenvalues and percent of variance accounted for:

FACTOR INFORMATION BASED ON CORRELATION MATRIX

		Eigenval	Percent	Cum Pct
Factor	1	3.3418	37.1313	37.1313
Factor	2	1.7543	19.4918	56.6230
Factor	3	1.5441	17.1563	73.7794
Factor	4	0.4513	5.0140	78.7933
Factor	5	0.4121	4.5787	83.3720
Factor	б	0.3976	4.4175	87.7896
Factor	7	0.3841	4.2683	92.0579
Factor	8	0.3688	4.0982	96.1561
Factor	9	0.3460	3.8439	100.0000

Figure 6.1 presents the associated scree plot:



FIGURE 6.1. Scree Plot

The line on the plot labeled PC flattens at factor 4. The general trend suggests a three factor solution. The line labeled FA is for factor extraction using the reduced correlation matrix (i.e., a correlation matrix with communalities in the diagonal). To make sense of it, we need to consider the output associated with the reduced correlation matrix:

FACTOR INFORMATION BASED ON REDUCED CORRELATION MATRIX

Factor 12.6470102.8980102.8980Factor 21.031140.0826142.9809Factor 30.818731.8263174.8069Factor 4-0.1268-4.9286169.8783Factor 5-0.1306-5.0754164.8029Factor 6-0.3350-13.0218151.7813Factor 7-0.3927-15.2657136.5154Factor 8-0.4354-16.9263119.5893Factor 9-0.5039-19.5892100.0000			Eigenval	Percent	Cum Pct
Factor 2 1.0311 40.0826 142.980 Factor 3 0.8187 31.8263 174.806 Factor 4 -0.1268 -4.9286 169.878 Factor 5 -0.1306 -5.0754 164.802 Factor 6 -0.3350 -13.0218 151.781 Factor 7 -0.3927 -15.2657 136.5154 Factor 8 -0.4354 -16.9263 119.5892 Factor 9 -0.5039 -19.5892 100.0004	Factor	1	2.6470	102.8980	102.8980
Factor 3 0.8187 31.8263 174.8069 Factor 4 -0.1268 -4.9286 169.8783 Factor 5 -0.1306 -5.0754 164.8029 Factor 6 -0.3350 -13.0218 151.7813 Factor 7 -0.3927 -15.2657 136.5154 Factor 8 -0.4354 -16.9263 119.5893 Factor 9 -0.5039 -19.5892 100.0000	Factor	2	1.0311	40.0826	142.9805
Factor 4 -0.1268 -4.9286 169.8783 Factor 5 -0.1306 -5.0754 164.8023 Factor 6 -0.3350 -13.0218 151.7813 Factor 7 -0.3927 -15.2657 136.5154 Factor 8 -0.4354 -16.9263 119.5893 Factor 9 -0.5039 -19.5892 100.0004	Factor	3	0.8187	31.8263	174.8069
Factor 5 -0.1306 -5.0754 164.802 Factor 6 -0.3350 -13.0218 151.781 Factor 7 -0.3927 -15.2657 136.515 Factor 8 -0.4354 -16.9263 119.589 Factor 9 -0.5039 -19.5892 100.0000	Factor	4	-0.1268	-4.9286	169.8783
Factor 6 -0.3350 -13.0218 151.781 Factor 7 -0.3927 -15.2657 136.5154 Factor 8 -0.4354 -16.9263 119.5893 Factor 9 -0.5039 -19.5892 100.0000	Factor	5	-0.1306	-5.0754	164.8029
Factor 7 -0.3927 -15.2657 136.515 Factor 8 -0.4354 -16.9263 119.5892 Factor 9 -0.5039 -19.5892 100.0000	Factor	6	-0.3350	-13.0218	151.7811
Factor 8-0.4354-16.9263119.5892Factor 9-0.5039-19.5892100.0000	Factor	7	-0.3927	-15.2657	136.5154
Factor 9 -0.5039 -19.5892 100.000	Factor	8	-0.4354	-16.9263	119.5892
	Factor	9	-0.5039	-19.5892	100.0000

The eigenvalues for the correlation matrix reflect the variance accounted for by each factor. The eigenvalues for the *reduced* correlation matrix, by contrast, reflect the *shared* variance accounted for by each factor. In the present case, a one factor model was fit, so all of the shared variance must lie in the first factor. This is why the first entry under the column for percent of (shared) variance equals 100% (it is 102% because of rounding). Here is the same table when we fit a two factor model instead of a one factor model:

		Eigenval	Percent	Cum Pct
Factor	1	2.8115	68.6398	68.6398
Factor	2	1.3468	32.8801	101.5199
Factor	3	0.8491	20.7308	122.2507
Factor	4	0.0100	0.2437	122.4944
Factor	5	-0.0067	-0.1641	122.3303
Factor	6	-0.0150	-0.3671	121.9633
Factor	7	-0.0303	-0.7408	121.2225
Factor	8	-0.4091	-9.9867	111.2358
Factor	9	-0.4602	-11.2358	100.0000

FACTOR INFORMATION BASED ON REDUCED CORRELATION MATRIX

Note that the solution reaches 100% explained shared variance at two factors, with the first factor accounting for about 68% of the shared variance and the second factor accounting for about 32% of the shared variance. Here is the same table when we fit a three factor model to the data:

FACTOR INFORMATION BASED ON REDUCED CORRELATION MATRIX

		Eigenval	Percent	Cum Pct
Factor	1	2.9486	53.9485	53.9485
Factor	2	1.3697	25.0592	79.0077
Factor	3	1.1474	20.9923	99.9999
Factor	4	0.0409	0.7483	100.7482
Factor	5	0.0158	0.2897	101.0379
Factor	б	0.0056	0.1023	101.1401
Factor	7	-0.0030	-0.0547	101.0854
Factor	8	-0.0198	-0.3624	100.7231
Factor	9	-0.0395	-0.7231	100.0000

The solution reaches 100% explained shared variance at three factors, with the first factor accounting for about 54% of the shared variance, the second factor accounting for about 25% of the shared variance, and the third factor accounting for about 21% of the shared variance. Here is the table for a four factor solution:

FACTOR INFORMATION BASED ON REDUCED CORRELATION MATRIX

		Eigenval	Percent	Cum Pct
Factor	1	2.9777	52.1785	52.1785
Factor	2	1.3784	24.1540	76.3325
Factor	3	1.1909	20.8685	97.2011
Factor	4	0.1597	2.7988	99.9999
Factor	5	0.0181	0.3166	100.3165
Factor	б	0.0059	0.1026	100.4191
Factor	7	-0.0001	-0.0020	100.4171
Factor	8	-0.0023	-0.0403	100.3769
Factor	9	-0.0215	-0.3769	100.0000

The solution reaches 100% explained shared variance at four factors, but note the small amount of the shared variance accounted for by the fourth factor (2.8%). This suggests a three factor solution, which is consistent with the analysis of the original correlation matrix (the PC line in the scree plot).

The output also includes a formal parallel analysis (Zwick & Velicer, 1986) to decide the number of factors. Here are the results for it:

```
NUMBER OF FACTORS BASED ON PARALLEL ANALYSIS
Number of factors: 3
```

The parallel analysis also suggests a three factor model.

We next conduct analyses for a one factor model, a two factor model, a three factor model, and a four factor model so we can compare fit indices for each model (see the factor analysis primer for details). Here are the chi square tests of model fit for each model, taken from each of the four outputs:

```
One factor model:Chi square = 1367.4df = 27p < 0.05Two factor model:Chi square = 610.3df = 19p < 0.05Three factor model:Chi square = 9.36df = 12p = 0.67Four factor model:Chi square = 2.09df = 6p = 0.91
```

Both the one factor and two factors models yield statistically significant and, hence, poor model fits. The three and four factor models yield non-statistically significant results, which are consistent with good model fit.

Preacher et al. (2013) recommend using the RMSEA fit statistic and its 90% confidence interval to make decisions about the number of factors to retain. Starting with a one factor model, we successively increase the number of factors until we find the first model that has a lower bound confidence interval value less than the traditional close fit RMSEA standard of 0.05. This model is the number of factors to retain (see the primer on factor analysis for details). Here are the relevant statistics, gathered from the different outputs:

```
One factor model: RMSEA = 0.26, 90% CI = 0.25 to 0.27
Two factor model: RMSEA = 0.20, 90% CI = 0.19 to 0.22
Three factor model: RMSEA = 0.00, 90% CI = 0.00 to 0.037
Four factor model: RMSEA = 0.00, 90% CI = 0.00 to 0.047
```

The first model where the lower bound CI for the RMSEA is less than 0.05 is a three factor model. Indeed, even the upper bound CI of the three factor model (0.037) is less than the traditional RMSEA standard of 0.05 for defining close fit, so the three factor model is quite viable.

We also can perform significance tests of incremental fit for successively more complex models to provide perspectives on the appropriate number of factors to retain. Rather than using the traditional nested chi square difference test that evaluates a null hypothesis of zero incremental fit by a more complex model, we use instead the approach that tests for non-trivial (rather than 0) incremental fit (see the factor analysis primer for details). Specifically, we test for RMSEA differences greater than the close fit standard of 0.05 using the method of Liu and Bentler (2011) in the ASA program "Structural Equation Modeling > Model Fit and Model Comparison > RMSEA for nested models with close fit null hypothesis: Liu and Bentler." Here are the results comparing the two factor model with the one factor model:

RMSEA null equivalence standard: 0.05 Sample size: 750 Unconstrained model degrees of freedom: 19 Constrained model degrees of freedom: 27 Traditional chi square difference between models: 757.02 RESULTS RMSEA standard: 0.05 Model df difference: 8 p value for incremental fit against standard: 0.00000

The comparison is statistically significant (p < 0.05), indicating that a two factor model improves fit over a one factor by more than an RMSEA difference standard of 0.05. This comparison also was statistically significant for a three factor model compared to a two factor model (p < 0.05) but not for a four factor model relative to a three factor model (p = 0.99). Thus, significance tests of non-trivial incremental fit using differences in RMSEAs favor the three factor model.

Here are the HBIC values for the four models:

HBIC for one factor model:1188.67HBIC for two factor model:484.56HBIC for three factor model:-70.08HBIC for four factor model:-37.63

For HBICs, we favor models with the lowest HBIC, which is the three factor model. The three factor model is superior to the other models by a substantial margin (see the primer on information theory fit indices for details).

We also can compute the comparative fit index (CFI) to determine the proportion of incremental fit that a given model yields relative to the model with one factor less. We use the ASA program "Structural Equation Modeling > Model Fit and Model Comparison > CFI comparing two models." Here are the results, summarized across the

three model difference analyses:

```
CFI for two versus one factor model: 0.56
CFI for three versus two factor model: 1.00
CFI for four versus three factor model: 0.00
```

The two factor model improved prediction over the one factor model by 56% and the three factor model improved prediction over the two factor model by 100% or more. The four factor model did not improve prediction over the three factor model, after taking into account model complexity as reflected by the model degrees of freedom.

Finally, we can compare the models in terms of their average (root mean square) discrepancy between predicted and observed correlations. Here are the results, summarized across the four runs:

One factor model average disparity: 0.191 Two factor model average disparity: 0.130 One factor model average disparity: 0.007 One factor model average disparity: 0.003

The disparity drops dramatically for the three factor model relative to the one and two factor models and only trivially improves for the four factor model.

Everything points to a three factor model. This type of convergence will not always occur. Sometimes different tests will lead to different conclusions. When this happens, we tend to give greater weight to the less subjective methods that are accommodating of sampling error, such as the Preacher et al. (2013) strategy based on RMSEAs. We also place a premium on factor interpretability, preferring solutions that make the most conceptual sense.

Model Fit

Given a three factor model, it is useful to examine indices of how well that model fits the data. Here is the relevant output:

```
MODEL FIT INFORMATION
(-9999 means could not be computed)
Root mean square of off-diagonal correlation residuals: 0.007381
RMSEA: 0.0
90% confidence interval: .0000 to .0372
Lower and upper margin of error: 0.00000, 0.03720
Haughton Bayesian Information Criterion: -70.083105
95% confidence interval: -9999 to -68.7360
Lower and upper margin of error: -9999, 1.34709
```

```
Tucker-Lewis index: 1.003166
Comparative fit index: 1.0
Model chi square: 9.357773
Model df: 12
Model p value: 0.672106
Model objective (fit function): 0.012592
Null model chi square: 2546.388
Null model df: 36
Null model RMSEA: 0.305125
```

The average absolute disparity between predicted and observed correlations was only 0.007. The RMSEA was 0 with a 90% confidence interval of 0 to 0.037. Even the upper limit of the confidence interval is less than the traditional standard for a close fitting model (0.05). The comparative fit index (CFI) was 1.00, which exceeds the traditional standard of a good fitting model of 0.95 or greater. The chi square test for model fit was statistically non-significant (chi square = 9.36, df = 12, p < 0.68), which also is consistent with a good model fit. Everything points to a good fitting model. (As an aside, Kenny et al. (2015) discourage the use of the RMSEA statistic if the RMSEA for the null model is less than 0.16. The RMSEA for the null model was 0.31).

We also examine the residual correlation matrix to see if there are any specific correlations that were not well reproduced by the three factor model. Here is the output:

		V1	V2	V3	V4	V5	VG
PTSD1	(V1)	-	-0.0001	-0.0003	0.0043	0.0029	-0.0050
PTSD2	(V2)	-0.0001	-	0.0005	-0.0111	0.0050	0.0049
PTSD3	(V3)	-0.0003	0.0005	_	0.0042	-0.0060	0.0008
PTSD4	(V4)	0.0043	-0.0111	0.0042	-	-0.0005	-0.0005
PTSD5	(V5)	0.0029	0.0050	-0.0060	-0.0005	-	0.0009
PTSD6	(V6)	-0.0050	0.0049	0.0008	-0.0005	0.0009	_
PTSD7	(V7)	-0.0028	0.0020	0.0011	-0.0217	0.0151	0.0056
PTSD8	(V8)	0.0096	-0.0018	-0.0078	0.0176	-0.0163	-0.0013
PTSD9	(V9)	-0.0085	-0.0001	0.0082	0.0057	0.0007	-0.0053
		V7	V8	V9			
PTSD1	(V1)	-0.0028	0.0096	-0.0085			
PTSD2	(V2)	0.0020	-0.0018	-0.0001			
PTSD3	(V3)	0.0011	-0.0078	0.0082			
PTSD4	(V4)	-0.0217	0.0176	0.0057			
PTSD5	(V5)	0.0151	-0.0163	0.0007			
PTSD6	(V6)	0.0056	-0.0013	-0.0053			
PTSD7	(V7)	-	0.0000	0.0004			
PTSD8	(V8)	0.0000	-	-0.0004			
ptsd9	(V9)	0.0004	-0.0004	_			

RESIDUAL MATRIX FOR CORRELATIONS

No single disparity is particularly noteworthy; all of them are quite low.

Interpretation of the Model

Here are the (geomin) rotated factor loadings for the model:

ROTATED	FACTOR	LOADINGS	(PATTERN	MATRIX)
	F1	F2	F3	
PTSD1	0.0163	3 0.781		118
PTSD2	0.0016	5 0.776	51 -0.0	025
PTSD3	-0.0074	1 0.780	0.02	281
PTSD4	0.0149	0.013	B7 0.7	503
PTSD5	0.0138	3 0.006	54 0.74	411
PTSD6	-0.0137	7 -0.006	51 0.83	110
PTSD7	0.8111	-0.041	-0.02	201
PTSD8	0.7890	0.029	99 -0.0	035
PTSD9	0.7402	2 0.022	27 0.04	458

Before interpreting the loadings, we want to gain a sense of their margins of error (MOEs). Here is the output that shows the margins of error for the loadings for each variable for the first factor:

FACTOR LOADING CONFIDENCE INTERVALS AND MARGINS OF ERROR FACTOR 1, PTSD1 Loading: 0.016337 95% IJK confidence interval: -.0294 to .0621 Lower and upper margin of error: -0.04576, 0.04576 FACTOR 1, PTSD2 Loading: 0.001633 95% IJK confidence interval: -.0488 to .0520 Lower and upper margin of error: -0.05040, 0.05040 FACTOR 1, PTSD3 Loading: -0.007408 95% IJK confidence interval: -.0574 to .0426 Lower and upper margin of error: -0.05004, 0.05004 FACTOR 1, PTSD4 Loading: 0.014899 95% IJK confidence interval: -.0375 to .0673 Lower and upper margin of error: -0.05237, 0.05237 FACTOR 1, PTSD5 Loading: 0.013832

```
95% IJK confidence interval: -.0365 to .0642
  Lower and upper margin of error: -0.05037, 0.05037
FACTOR 1, PTSD6
  Loading: -0.013729
   95% IJK confidence interval: -.0585 to .0310
  Lower and upper margin of error: -0.04475, 0.04475
FACTOR 1, PTSD7
  Loading: 0.811091
   95% IJK confidence interval: .7630 to .8592
  Lower and upper margin of error: -0.04808, 0.04808
FACTOR 1, PTSD8
   Loading: 0.789041
   95% IJK confidence interval: .7456 to .8325
  Lower and upper margin of error: -0.04346, 0.04346
FACTOR 1, PTSD9
   Loading: 0.740182
   95% IJK confidence interval: .6932 to .7872
   Lower and upper margin of error: -0.04703, 0.04703
```

The MOEs are all near 0.05, which is quite reasonable. This also was the case for the two other factors as well.

The strongest loadings for the first factor were for ptsd7 (0.81), ptsd8 (0.79), and ptsd9 (0.74). The variables focus on (a) excessive worrying, (b) difficulty concentrating, and (c) feeling jumpy and easily startled. The latent factor that contributes to the correlations between these variables might be some form of generalized anxiety that results from the stressful event(s). The strongest loadings for the second factor were for ptsd1 (0.78), ptsd2 (0.78), and ptsd3 (0.78). The variables are (a) upsetting memories from the past event, (b) nightmares, and (c) intense reactions to reminders of the event. The latent factor that contributes to the correlations between these variables might be some form of generalized negative affect associated with reliving the event. The strongest loadings for the third factor were for ptsd4 (0.75), ptsd5 (0.74), and ptsd6 (0.81). The variables are (a) avoiding activities that might remind one of the event, (b) loss of interest in activities and life in general, and (c) feeling emotionally numb. The latent factor that contributes to the correlations between these variables might be some form of generalized avoidance and numbing. Thus, three general reactions to experiencing stressful war events are (1) generalized anxiety, (2) negative affect from reliving the event, and (3) generalized avoidance and numbing. These three general reactions manifest themselves into the nine specific symptoms, but each symptom also is shaped by unique factors that are unrelated to these three generalized responses. The proportion of unique and common variance for each of the nine symptoms are:

Communality Uniqueness 0.6106 0.3894 PTSD1 0.6017 0.3983 PTSD2 0.3777 PTSD3 0.6223 PTSD4 0.5770 0.4230 PTSD5 0.5587 0.4413 PTSD6 0.6483 0.3517 PTSD7 0.6341 0.3659 PTSD8 0.6339 0.3661 PTSD9 0.5790 0.4210

VARIABLE COMMUNALITIES AND UNIQUENESS

Each symptom has about 40% unique variance and 60% common variance. As noted in the primer on factor analysis, the unique variance is substantial and we probably should focus theorizing on it as much as on the three generalized reactions that account for 60% of the variance in each symptom.

Finally, we are interested in the correlations between the factors. Here is the relevant output:

FACTOR CORRELATIONS

	Fl	F2	F3
F1	1.0000	0.2559	0.2830
F2	0.2559	1.0000	0.3495
F3	0.2830	0.3495	1.0000

and the relevant margins of errors:

FACTOR CORRELATION CONFIDENCE INTERVALS AND MARGINS OF ERROR

FACTOR 1 WITH FACTOR 2

Correlation: 0.2559 95% IJK confidence interval: .1712 to .3368 Lower and upper margin of error: -0.08468, 0.08093

FACTOR 1 WITH FACTOR 3

Correlation: 0.283 95% IJK confidence interval: .1989 to .3630 Lower and upper margin of error: -0.08414, 0.07999

FACTOR 2 WITH FACTOR 3

Correlation: 0.3495

```
95% IJK confidence interval: .2741 to .4207
Lower and upper margin of error: -0.07541, 0.07114
```

The three generalized reactions to stress are correlated about 0.25 to 0.35, with margins of error of about \pm 0.08. All of the correlations are statistically significant (p < 0.05) because their confidence intervals do not include the value of zero. The confidence intervals are based on Fisher r to Z transformations with back-transformations.

Writing it Up

Because of space limitations in journals, we do not have room to describe the preliminary analyses, but we would indicate in the Method analytic section the general strategies we used for preliminary analyses and report that the analyses affirmed the use of maximum likelihood factor analysis. We also would explain in that section our strategy of using a confidence interval to define margins of error and how the margins of error are represented (e.g., "Margins of errors (MOEs) are calculated from 95% confidence intervals and are the absolute distance between the lower limit or upper limit of the interval minus the parameter estimate, whichever is larger, unless otherwise noted"). Here is how we might write-up the results:

"A maximum likelihood factor analysis was performed followed by an oblique geomin rotation. The first five eigenvalues and their associated percents of variance accounted for were 3.34 (37.1%), 1.75 (19.5%), 1.54 (17.2%), 0.45 (5.0%), and 0.41 (4.6%). Both a scree test and a parallel analysis (Zwick and Velicer, 1986) suggest 3 factors should be retained. Table 1 presents more formal analyses regarding the number of factors, all of which support a 3 factor model. Preacher et al. (2013) recommend using the RMSEA fit statistic and its 90% confidence interval to choose the number of factors: Starting with a one factor model, one successively increases the number of factors until one identifies the first model that has a lower bound confidence interval value less than the traditional close fit RMSEA standard of 0.05. As seen in Table 1, this was a model with 3 factors. The three factor model also had the lowest HBIC (see Table 1). The root mean square residual between predicted and observed correlations was large for the one and two factor models (0.19 and 0.13, respectively), but satisfactory for the three factor model (0.007). The four factor model improved the disparity index trivially (0.003). A nested CFI statistic was computed comparing a given model with a model with one less factor. There was substantial improvement in fit for successive models through the three factor model but not for the four factor model (see Table 1). Finally, a test of incremental model fit based on an RMSEA increment of 0.05 when comparing a given model with a model with one less factor (Liu and Bentler, 2011) yielded a statistically significant (p < 0.05) increment in fit for the three factor model but not the for four factor model. All results suggest a three factor model is appropriate.

The fit of the three factor model was good. The average absolute disparity between predicted and observed correlations was 0.007. The RMSEA was < .001 with a 90% confidence interval of 0 to 0.037. The comparative fit index (CFI) was 1.00, which exceeds the traditional standard of a good fitting model of 0.95 or greater. The chi square test for model fit was statistically non-significant (chi square = 9.36, df = 12, p < 0.68), which also is consistent with good model fit. There were no specific correlations that were not well reproduced by the model.

The standardized rotated factor loadings and their margins of error (MOEs) are presented in Table 2. The MOEs are based on infinitesimal jackknife methods (Zhang et al., 2012) and all are near \pm 0.05. The strongest loadings for the first factor were the variables focused on (a) excessive worrying (0.81), (b) difficulty concentrating (0.79)and (c) feeling jumpy and easily startled (0.74). The latent factor that contributes to the correlations between these variables might be some form of generalized anxiety that results from the stressful event(s). The strongest loadings for the second factor were for the variables (a) upsetting memories from the past event (0.78), (b) nightmares (0.78), and (c) intense reactions to reminders of the event (0.78). The latent factor that contributes to the correlations between these variables might be some form of generalized negative affect associated with reliving the event. The strongest loadings for the third factor were for (a) avoiding activities that might remind one of the event (0.75), (b) loss of interest in activities and life in general (0.74), and (c) feeling emotionally numb (0.81). The latent factor that contributes to the correlations between these variables might be some form of generalized avoidance and numbing. Thus, three general reactions to experiencing stressful war events are (1) generalized anxiety, (2) negative affect from reliving the event, and (3) generalized avoidance and numbing. These three general reactions manifest themselves into the nine specific symptoms, but each symptom also is shaped by unique factors that are unrelated to the generalized responses. The proportion of unique variance for each symptom variable is in Table 2. Each symptom has about 40% unique variance and 60% common variance.

The estimated factor correlations were 0.26 ± 0.08 between generalized anxiety (GA) and generalized negative affect with reliving the event (GNARE), 0.28 ± 0.08 between GA and generalized avoidance and numbing (GAN), and 0.35 ± 0.08 between GNARE and GAN. All three correlations were statistically significant (p < 0.05)."

	Model	Model	Model	Corr	Nested	Nested
Model	Chi square	<u>RMSEA</u>	<u>HBIC</u>	Disparity	<u>CFI</u>	<u>RMSEA</u>
One factor	1367.4* (27)	0.26 (0.25, 0.27)	1188.7	0.191	-	-
Two factors	610.3* (19)	0.20 (0.19, 0.22)	484.6	0.130	0.56	p < 0.05
Three factors	9.36 (12)	0.00 (0.00, 0.04)	-70.1	0.007	1.00	p < 0.05
Four factors	2.09 (6)	0.00 (0.00, 0.05)	-37.6	0.003	0.00	p = 0.99

Table 1: Statistics for Determining the Number of Factors

Notes: * p < 0.05; Statistically non-significant chi squares are consistent with good model fit (entries in parentheses are the model degrees of freedom); RMSEAs less than 0.08 are consistent with good model fit (entries in parentheses are 90% confidence intervals). HBIC is the Haughton Bayesian Information Criterion (lower values indicate better model fit); Corr disparity is the root mean square disparity between predicted and observed correlations; Nested CFI is a comparative fit index comparing the model with a model with one less factor; Nested RMSEA is a p value for the test of incremental fit relative to a model with one less factor using an increment of 0.05 for the null hypothesis

Variable	Factor 1	Factor 2	Factor 3	<u>Uniqueness</u>
Upsetting memories	0.02 ± 0.05	0.78 ± 0.04	-0.01 ± 0.04	0.39
Nightmares	0.00 ± 0.05	0.78 ± 0.04	0.00 ± 0.05	0.40
Intense reactions to reminders	$\textbf{-0.01} \pm 0.05$	0.78 ± 0.04	0.03 ± 0.05	0.38
Avoid activities	0.01 ± 0.05	0.01 ± 0.05	0.75 ± 0.05	0.42
Loss of interest	0.01 ± 0.05	0.01 ± 0.05	0.74 ± 0.05	0.44
Emotionally numb	$\textbf{-0.01} \pm 0.04$	$\textbf{-0.01} \pm 0.04$	0.81 ± 0.05	0.35
Excessive worrying	0.81 ± 0.05	$\textbf{-0.04} \pm 0.05$	-0.02 ± 0.04	0.37
Difficulty concentrating	0.79 ± 0.04	0.03 ± 0.05	0.00 ± 0.04	0.37
Jumpy and easily startled	0.74 ± 0.05	0.02 ± 0.05	0.05 ± 0.05	0.42

Table 2: Factor Loadings for Three Factor Model

Notes: Margins of error are reported for each loading; Uniqueness is the proportion of unique variance in each measure; One minus uniqueness is the communality

REFERENCES

Curran, J. M. (2005). An introduction to Bayesian credible intervals for sampling error in DNA profiles. *Law, Probability and Risk*, 4, 115-126.

Kenny, D. A., Kaniskan, B., & McCoach, D. B. (2015). The performance of RMSEA in models with small degrees of freedom. *Sociological Methods and Research*, 44, 486-507.

Liu, L. & Bentler, P. (2011). Quantified choice of RMSEAs for evaluation and power analysis of small differences between structural equation models. *Psychological Methods*, 16, 116–126.

Preacher, K., Zhang, G., Kim, C. & Mels, G. (2013). Choosing the optimal number of factors in exploratory factor analysis: A model selection perspective. *Multivariate Behavioral Research*, 48, 28–56.

Zhang, G., Preacher, K. J., & Jennrich, R. I. (2012). The infinitesimal jackknife with exploratory factor analysis. *Psychometrika*, 77, 634-648.

Zwick, W.R. & Velicer, W.F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin*, 99, 432-442.